



# A model for droplet heating and its implementation into ANSYS Fluent<sup>☆</sup>



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## ABSTRACT

The main ideas of the model for droplet heating and evaporation, based on the analytical solution to the heat conduction equation inside the droplet, and its implementation into ANSYS Fluent are described. The model is implemented into ANSYS Fluent using User-Defined Functions (UDF). The predictions of ANSYS Fluent with the new model are verified against the results predicted by in-house research code for an *n*-dodecane droplet heated and evaporated in hot air. Also, the predictions of this version of ANSYS Fluent are compared with in-house experimental data.

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## 1. Introduction

A new model for multi-component droplet heating and evaporation, based on the analytical solutions to the heat transfer and species diffusion equations, has been developed by our group (see Refs. [1,2] for the details). This model has been validated based on the available experimental data and the predictions of the numerical codes using the analytical solution to these equations [3,4].

In the current study the analysis is restricted to mono-component droplets. The main ideas of the new model and the results of its implementation into the commercial CFD code ANSYS Fluent, via User-Defined Functions (UDF), macros, supported by ANSYS Fluent, are summarised. The results of the implementation of the model are compared with the predictions of the in-house code and validated against in-house experimental data.

The mathematical formulation and the implementation of the model are described in Section 2. In Section 3.1, the predictions of ANSYS Fluent with the new model are verified against the predictions of the in-house code. In Section 3.2, the experimental set-up is described, and the results of simulations are compared with the

experimental data. The main results of the paper are summarised in Section 4.

## 2. Formulation of the problem

In the conventional approach, used in most available CFD codes, including ANSYS Fluent, droplet heating is modelled based on the solution to the following energy balance equation:

$$c_{pl}m_d \frac{dT}{dt} = 2\pi Nuk_g R_d (T_g - T_s) + L\dot{m}_d + q_{int}, \quad (1)$$

where  $c_{pl}$  is droplet liquid specific heat capacity,  $m_d$  and  $R_d$  are droplet mass and radius, respectively,  $Nu$  is the Nusselt number,  $k_g$  is gas thermal conductivity,  $T_g$  and  $T_s$  are gas and surface temperatures, respectively,  $L$  is the latent heat of evaporation,  $q_{int}$  is heat supplied or removed from internal sources (e.g. chemical reactions). The derivation of this equation is based on the assumption that the effects of temperature gradients inside droplets can be ignored. This assumption is commonly supported by the fact that liquid thermal conductivity is much higher than gas thermal conductivity in most engineering applications. At the same time, when modelling transient processes this assumption should be based on the comparison of the liquid and gas thermal diffusivities and the values of the Fourier number. In most engineering applications, including Diesel engines, liquid thermal diffusivities are much lower than

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### Nomenclature

$B_M, B_T$	Spalding mass and heat transfer numbers
$c_p$	specific heat capacity at constant pressure
$D$	binary diffusivity coefficient of vapour in air
$h$	convection heat transfer coefficient
$I_n$	integrals, used in series (2) and (6)
$j$	parameter, defined in Eq. (4b)
$k$	thermal conductivity
$L$	latent heat of evaporation
$m$	mass
$\dot{m}$	evaporation rate
$M$	molar mass
$N_L$	number of layers inside a droplet
$Nu$	Nusselt number
$Pe$	Peclet number
$p$	pressure
$Pr$	Prandtl number
$q$	heat flux
$r$	radial coordinate from the centre of the droplet
$R_d$	radius of a droplet
$Re$	Reynolds number
$Sc$	Schmidt number
$Sh$	Sherwood number
$t$	time
$T$	temperature
$\mathbf{v}$	velocity
$Y$	mass fraction
$x$	molar fraction

### Greek symbols

$\kappa$	parameter defined by Eq. (4a)
$\lambda_n$	eigenvalues defined by Eq. (3)
$\mu$	dynamic viscosity
$\rho$	density
$\phi$	parameter defined by Eq. (5e)
$\chi$	correction function defined by Eq. (4b)
$\zeta$	parameter defined by Eq. (4a)

### Subscripts

$d$	droplet
$eff$	effective
$g$	gas
$int$	internal
$l$	liquid
$ref$	reference value
$s$	surface of droplet
$sat$	saturation
$t$	total
$v$	vapour
$0$	value at the beginning of a time step
$\infty$	value in the far field

an analytical solution to this equation during any time step  $\Delta t$  ( $t \in [0, \Delta t]$ ) has been found in the form [2,4]:

$$T(r, t) = \frac{1}{r} \sum_{n=1}^{\infty} \left\{ \left( I_n - \frac{R_d \sin \lambda_n}{\lambda_n^2} \zeta(0) \right) \frac{\exp(-\kappa \lambda_n^2 t)}{b_n} - \frac{R_d \sin \lambda_n}{b_n \lambda_n^2} \int_0^t \frac{d\zeta(\tau)}{d\tau} \exp(-\kappa \lambda_n^2 (t - \tau)) d\tau \right\} \sin \left( \lambda_n \frac{r}{R_d} \right) + T_{eff}(t), \quad (2)$$

where  $\lambda_n$  are positive roots to the eigenvalue equation

$$\lambda \cos \lambda + j \sin \lambda = 0; \quad (3)$$

in ascending order,

$$b_n = \frac{1}{2} \left( 1 + \frac{j}{j^2 + \lambda_n^2} \right), \quad I_n = \int_0^{R_d} \frac{r}{R_d} T_0(r) \sin \left( \lambda_n \frac{r}{R_d} \right) dr,$$

$T_0(r)$  is the initial temperature distribution inside the droplet or the distribution predicted at the previous time step;

$$\kappa = \frac{k_{eff}}{c_{pl} \rho_l R_d^2}, \quad \zeta(t) = \frac{h T_{eff}(t) R_d}{k_{eff}}, \quad T_{eff} = T_g + \frac{\dot{m}_d L}{2\pi R_d Nu k_g}; \quad (4a)$$

$$j = \frac{h R_d}{k_{eff}} - 1, \quad h = \frac{k_g Nu}{2 R_d}, \quad k_{eff} = \chi k_l, \quad \chi = \left( 1.86 + 0.86 \tanh \left( 2.225 \lg \frac{Pe}{30} \right) \right), \quad (4b)$$

$$Pe = 0.79 |\mathbf{v}_g - \mathbf{v}_d| \frac{\mu_g}{\mu_l} \frac{Re_d^{1/3}}{1 + B_M} \frac{\rho_l R_d c_{pl}}{k_l}, \quad Re_d = \frac{2 R_d \rho_g |\mathbf{v}_g - \mathbf{v}_d|}{\mu_g}, \quad (4c)$$

where  $B_M = (Y_{vs} - Y_{v\infty}) / (1 - Y_{vs})$ ,  $Y_{v\infty}$  and  $Y_{vs}$  are mass fractions of vapour in the ambient gas and at the droplet surface. Note that

$$Y_{vs} = \frac{x_{vs} M_v}{x_{vs} M_v + (1 - x_{vs}) M_a}, \quad x_{vs} = \frac{p_{sat}}{p}.$$

The Nusselt number is approximated as [2]:

$$Nu = \frac{\ln(1 + B_T)}{B_T} Nu^*, \quad (5a)$$

$$Nu^* = 2 + \frac{(1 + Re_d Pr)^{1/3} \max(1, Re_d^{0.077}) - 1}{F(B_T)}, \quad (5b)$$

$$Sh^* = 2 + \frac{(1 + Re_d Sc)^{1/3} \max(1, Re_d^{0.077}) - 1}{F(B_M)}, \quad (5c)$$

$$F(B_{T,M}) = (1 + B_{T,M})^{0.7} \frac{\ln(1 + B_{T,M})}{B_{T,M}}, \quad B_T = (1 + B_M)^\phi - 1, \quad (5d)$$

$$\phi = \frac{c_{pv} \rho_g D}{k_g} \frac{Sh^*}{Nu^*}, \quad Pr = \frac{c_{pg} \mu_g}{k_g}, \quad Sc = \frac{\mu_g}{\rho_g D}. \quad (5e)$$

those of gas. This obviously brings into question the applicability of Eq. (1). This equation cannot be used at all when the Fourier numbers are small.

In an alternative approach to the problem of droplet heating, taking into account the effects of temperature gradient inside droplets, the transient heat conduction inside the droplet is solved subject to the boundary conditions at the surface of the droplet. Assuming that all the processes inside the droplet are spherically symmetric,

The term  $R_d \sin \lambda_n / (b_n \lambda_n^2) \int_0^t d\zeta(\tau) / d\tau \exp(-\kappa \lambda_n^2(t-\tau)) d\tau$  was shown to be negligibly small. This allowed us to simplify Expression (2) to:

$$T(r, t) = \frac{1}{r} \sum_{n=1}^{\infty} \left\{ \left( I_n - \frac{R_d \sin \lambda_n}{\lambda_n^2} \zeta(0) \right) \frac{\exp(-\kappa \lambda_n^2 t)}{b_n} \right\} \sin \left( \lambda_n \frac{r}{R_d} \right) + T_{\text{eff}}(t). \quad (6)$$

Droplet evaporation rate is estimated as [5]:

$$\dot{m}_d = -2\pi R_d D \rho_g \ln(1 + B_M) \text{Sh}^*. \quad (7)$$

### 3. Results and discussions

Eq. (7) was solved using the customised version of ANSYS Fluent; the right-hand side of this equation was calculated by the UDF. Thermodynamic parameters of a droplet were calculated based on the average temperature inside the droplet, which was calculated using Simpson's method [6]. Gas parameters in the vicinity of the droplet surface were calculated based on reference temperature:  $T_{\text{ref}} = (2T_s + T_g)/3$ . The roots to Eq. (3) were found using the bisection method with absolute accuracy of  $10^{-8}$ ; in simulations, we used 44 eigenvalues to calculate the series in Eq. (6).

The results predicted by ANSYS Fluent were verified against predictions made by the in-house code [4] (see Section 3.1) and then compared with experimental data (Section 3.2).

#### 3.1. Verification of the results

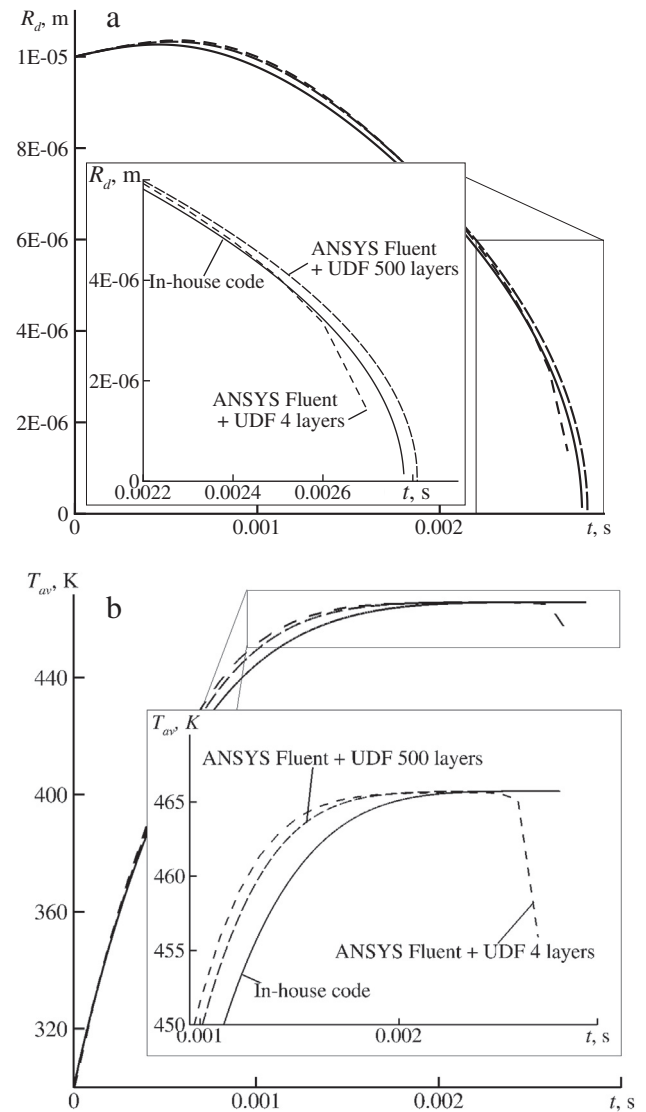
The verification was performed based on the analysis of heating and evaporation of an *n*-dodecane droplet of  $10 \mu\text{m}$  in radius with initial temperature of 300 K, uniform inside the droplet. The droplet was injected into quiescent air with  $T_a = 650 \text{ K}$ ,  $p_a = 101325 \text{ Pa}$ . The mass fraction of vapour away from the droplet was assumed to be negligible. The thermodynamic properties of *n*-dodecane were taken from Ref. [7]. The reference values were obtained using the in-house code [4]. This code, in its turn, was verified using the in-house code based on the analytical solution to the heat transfer equation inside the droplet (see Ref. [4] for the details). The droplet volume was discretised into  $N_L = 500$  concentric layers,  $\delta t = 10^{-6} \text{ s}$ ; the reference value for the evaporation time was taken equal to  $2.78 \cdot 10^{-3} \text{ s}$ .

Simulations using the built-in heating and evaporation model of ANSYS Fluent, Discrete Phase Model (DPM), showed that in this model the effect of droplet evaporation on droplet heating is overestimated. This resulted in low droplet temperature at the end of evaporation and high error in the prediction of evaporation time.

Relative errors in the evaporation times for a number of simulations of ANSYS Fluent + UDF with various  $\delta t$  and  $N_L$  are summarised in Table 1; 'large  $k_l$ ' corresponds to the case when droplet liquid thermal conductivity is set 1000 times higher in order to simulate

the case of uniform temperature distribution inside a droplet. *n*-Dodecane properties were introduced to ANSYS Fluent simulations via polynomial definition and DEFINE\_DPM\_PROPERTY UDFs.

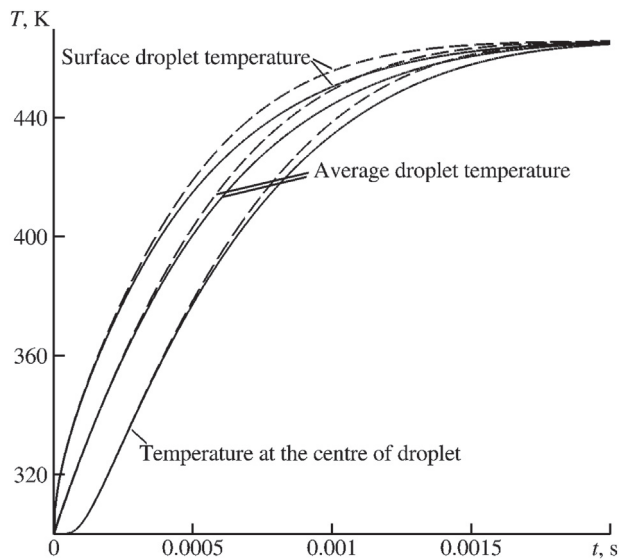
Case 1 corresponds to ANSYS Fluent + UDF simulations based on  $N_L = 500$ ,  $\delta t = 10^{-6} \text{ s}$ ; case 2:  $N_L = 4$ ,  $\delta t = 10^{-4} \text{ s}$ ; case 3: 'large  $k_l$ ',  $N_L = 100$ ,  $\delta t = 10^{-6} \text{ s}$ . As follows from Table 1,  $\delta t$  has a greater impact on the prediction of evaporation time than  $N_L$ . The smaller time step corresponds to the smaller values of the residuals; in most cases,  $\delta t = 10^{-4} \text{ s}$  leads to sufficiently accurate results. In three cases, denoted as 'N/A' in the table, the calculations did not converge; these correspond to coarse meshing. Overall, the results predicted by ANSYS Fluent + UDF based on Eq. (6) lead to errors of 1–5%, depending on  $N_L$  value. One per cent error at  $N_L = 500$  might be related to a different method of calculating integrals and eigenvalues (see Section 2 for the details) from the one used in the in-house code [4]. The results of the reference and ANSYS Fluent + UDF simulations for cases 1 and 2 are presented in Figs. 1 and 2. The time evolutions of droplet radii as predicted by ANSYS Fluent and in-house code are shown in Fig. 1a; Figs. 1b and 2 show the evolutions of the average



**Fig. 1.** *n*-Dodecane droplet heating and evaporation. Initial droplet temperature is 300 K, temperature of air is 650 K: (a) evolution of the droplet radius; (b) evolution of the droplet average temperature. Solid curves – in-house code, long dashed curves – case 1, dashed curves – case 2.

**Table 1**  
Relative errors of evaporation time predicted by ANSYS Fluent + UDF relative to the reference value of  $2.78 \cdot 10^{-3} \text{ s}$ , obtained using the in-house code [4].

$N_L \backslash \delta t, \text{ s}$	$10^{-4}$	$10^{-5}$	$10^{-6}$
500	$1.1 \cdot 10^{-2}$ (case 1)	$7.2 \cdot 10^{-3}$	$7.2 \cdot 10^{-3}$
100	$1.1 \cdot 10^{-2}$	$7.2 \cdot 10^{-3}$	$7.2 \cdot 10^{-3}$
50	$1.1 \cdot 10^{-2}$	$7.2 \cdot 10^{-3}$	$7.2 \cdot 10^{-3}$
10	N/A	$4.7 \cdot 10^{-2}$	$4.3 \cdot 10^{-2}$
4	N/A	N/A	$2.9 \cdot 10^{-2}$ (case 2)
Large $k_l$	$9 \cdot 10^{-3}$ (case 3)	$7.2 \cdot 10^{-3}$	$6.5 \cdot 10^{-2}$



**Fig. 2.** *n*-Dodecane droplet heating and evaporation. Initial droplet temperature is 300 K, temperature of air is 650 K: evolutions of the temperatures in the centre of the droplet, at the droplet surface, and of the average temperature. Solid curves – in-house code, long dashed curves – case 1.

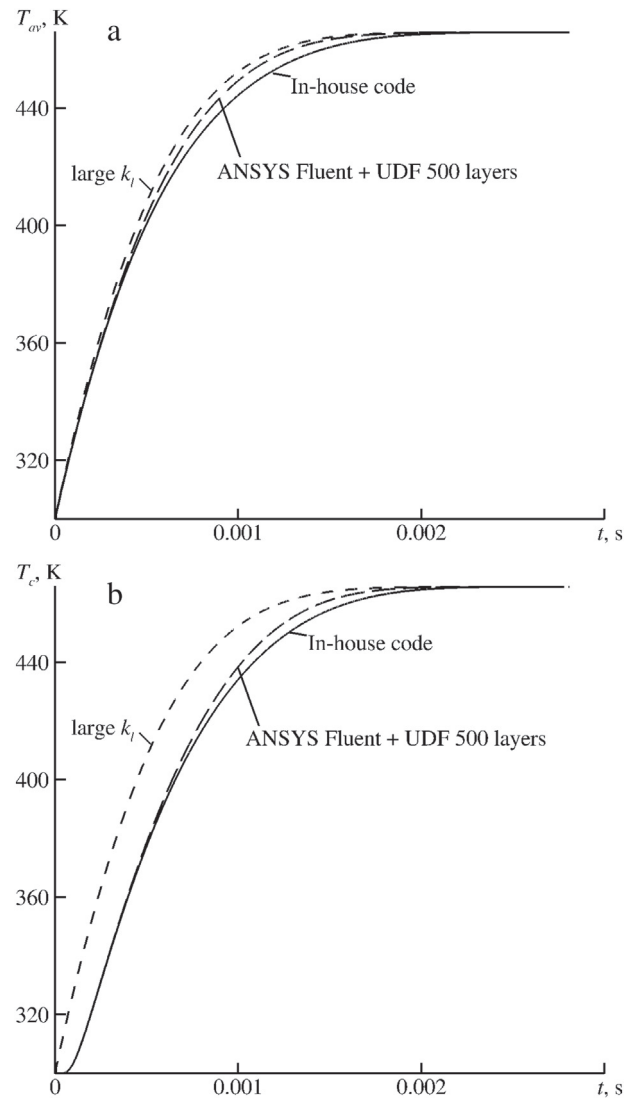
droplet temperature, temperatures in the centre, and at the surface of the droplet.

As follows from Figs. 1 and 2, the process of droplet heating and evaporation is modelled by ANSYS Fluent + UDF with satisfactory accuracy. The droplet radius increases at the initial stage due to swelling and then decreases due to evaporation. Note that no equations were added to the UDF in order to model droplet swelling: this is done using built-in ANSYS Fluent procedures, which take into account temperature-dependent density. In case 1, the predicted droplet temperatures increase monotonically; while in case 2, the droplet temperature drops slightly at the end of the evaporation process. This effect is believed to be related to the fact that the integrals  $I_n$  in Eq. (6) are calculated using coarse mesh (4 layers). The same effect takes place in the case of  $N_L = 10$ , and leads to higher errors or non-converging calculations (see Table 1). This should be taken into account when modelling a large number of droplets using ANSYS Fluent + UDF with coarse meshing inside droplets.

As follows from Fig. 2, the difference between temperatures at the droplet centre and at the droplet surface is high. This effect is ignored in conventional CFD calculations of droplet heating and evaporation in fuel spray modelling. The case of 'large  $k_i$ ' refers to uniform temperature distribution. As follows from Table 1, this does not have a big impact on evaporation time in the case considered. The average droplet temperature is also predicted with satisfactory accuracy (see Fig. 3a). However, the errors in predicted temperatures at the droplet centre are high (about 20 K) (see Fig. 3b). This behaviour is consistent with previous observations [8].

### 3.2. Comparison of the model with experimental data

Experiments were conducted using a preburn combustion vessel [9]. This constant-volume chamber is of cubic shape, offering six faces that can be fitted with transparent sapphire windows or metal ports. The thermodynamic conditions simulated inside the chamber are achieved after combustion of a preburn mixture of combustible gases composed of  $C_2H_2$ ,  $H_2$ ,  $N_2$  and  $O_2$ . After the spark-ignited preburn combustion, the internal temperature and pressure conditions decrease due to heat transfer to the vessel walls during a cool-down period of several seconds. The fuel is injected when the desired ambient conditions are reached during the cool-down.



**Fig. 3.** *n*-Dodecane droplet heating and evaporation. Initial droplet temperature is 300 K, temperature of air is 650 K: (a) evolution of the droplet average temperature; (b) evolution of the temperature in the centre of the droplet. Solid curves – in-house code, long dashed curves – case 1, dashed curves – case 3.

Sprays of *n*-dodecane were injected at a pressure of 50 MPa into a quiescent gas mixture,  $T_g = 707$  K,  $p_g = 6.16$  MPa, using a commercial common-rail solenoid-actuated injector equipped with a single-hole nozzle. The axially-drilled orifice located at the tip of the nozzle was of 0.180 mm diameter and featured a converging conical shape (KS 1.5). We selected a single-hole axially-drilled nozzle to ensure that the fuel spray would develop along the centre of the combustion vessel, where the thermodynamic conditions are best controlled. The fuel injector was temperature-controlled to ensure that the state of the fuel prior to injection was known, with a target temperature of 363 K. The nozzle's temperature was measured using a dummy injector fitted with a thermocouple. This thermocouple was used to adjust the injector cooling system, as described in Ref. [10]. For these operating conditions the maximum heating of the fuel due to friction inside the nozzle orifice is estimated at 20 K [11], hence the actual fuel temperature at the nozzle exit is expected to be  $370 \pm 10$  K.

The spray was back-illuminated through one sapphire window along a line of sight and imaged through another sapphire window on the other side. The optical system was composed of a

long-working-distance microscope lens, a high-speed CMOS camera (Photron SA-X2) and purpose-built LED illumination [12]. Two individual droplets were tracked throughout the video sequence, and their sizes measured based on the image processing technique described in Ref. [13]. The uncertainties concerning the droplet formation time and the droplet diameter were estimated as  $\pm 0.033$  ms, and  $\pm 10\%$ , respectively. The gas phase velocity in the vicinity of the droplet could not be measured, hence the relative droplet velocity is not known. However we can estimate an upper limit for the relative velocity from the observation that these droplets were not showing any sign of deformation, hence the gaseous Weber number for these conditions should be less than unity. Based on this observation we estimate that the relative initial droplet velocity should be less than 3.5 m/s. The mass fraction of vapour away from the droplets could not be measured. A steady-state analysis of the entrainment of the surrounding gas into a single free spray shows that there is no significant backward flow of vaporised fuel into the near-nozzle region of the spray [14]. Furthermore, our measurements were performed at the end of injection, when the entrainment of ambient gases into the jet increases significantly due to a strong entrainment wave, and the near-nozzle region becomes more fuel-lean than the steady jet [15,16].

There were no observations of the initial stages of droplet heating and evaporation processes. Hence, the comparison shown below will allow us to identify similarities in trends between the model and experimental data (to perform functionality testing of the model), but not quantitative agreement between the prediction of the model and experimental data. Three parameters could be used to adjust the model within the uncertainties of the experimental data: initial droplet size (to take into account droplet swelling); the initial relative droplet velocities; and the droplet initial temperature (360–380 K).

A series of simulations were performed with various  $R_{d0}$ ,  $T_{d0}$ ,  $v_0$ . Some results and comparison between experimental data and numerical simulation are presented in Fig. 4. Experimental data related to both droplets are shown by triangles. As follows from Fig. 4, the initial droplet velocity strongly affects the droplet evaporation time. In numerical simulations, droplets were injected into quiescent hot gas with initial velocity of  $v_0$ . As droplets evaporated, their relative velocities rapidly relaxed to zero. In the case of the initial droplet velocity of  $v_0 = 3.5$  m/s, at  $t = 2.983 \cdot 10^{-3}$  s, its velocity reduced to  $\sim 2.4 \cdot 10^{-2}$  m/s ( $R_{d0} = 17.5 \mu\text{m}$  and  $T_{d0} = 360$  K). In experiments, droplets were entrained in the air flow with complex vortex structures, although these were not captured. The velocity magnitude was calculated with a good accuracy, however the comparison between the experimental and theoretical data for droplet velocities could not be performed. The initial droplet temperature also has an impact on the evaporation time, the difference between cases  $T_0 = 360$  K and  $T_0 = 380$  K being about 4%.

As one can see from Fig. 4, the model predicts the same trends as observed in the experiments. It is not possible, however, to use these experimental data for the detailed validation of the model under consideration, compared with the conventional model based on the assumption that the liquid thermal conductivity is infinitely large. This is related both to the errors in experimental data and limitations of the model. The model is based on a number of simplifying assumptions (although some assumptions of the conventional model have been relaxed) and is not expected to be reliable for conditions close to the critical conditions considered in the experiments.

#### 4. Conclusion

The model for droplet heating and evaporation, based on the analytical solution to the heat conduction equation inside the droplet, has been implemented into ANSYS Fluent via User-Defined Functions

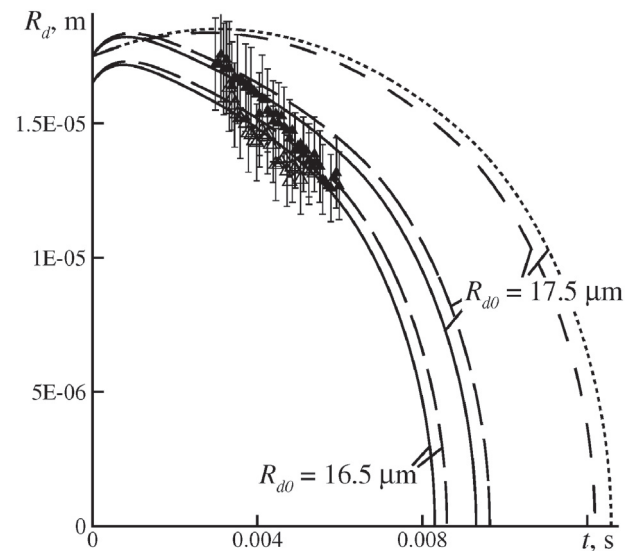


Fig. 4. *n*-Dodecane droplet heating and evaporation. Experimental data vs ANSYS Fluent predictions. Triangles – experimental data: filled symbols – droplet 1, hollow symbols – droplet 2. Solid curves show theoretical predictions for  $T_{d0} = 380$  K,  $v_0 = 3.5$  m/s; long dashed curves –  $T_{d0} = 360$  K,  $v_0 = 3.5$  m/s; dashed curves –  $T_{d0} = 380$  K,  $v_0 = 0$  m/s; dotted curves –  $T_{d0} = 360$  K,  $v_0 = 0$  m/s.

(UDF). Predictions of *n*-dodecane droplet heating and evaporation using ANSYS Fluent with the new model have been shown to be noticeably different from, and better than, the predictions of the standard Fluent code, demonstrating the need to adopt this new modelling process. The customised version of Fluent has been verified by comparing results for an evaporating *n*-dodecane droplet with the prediction of the in-house research code. Also, the predictions of this version of Fluent have been compared with the trends of the in-house experimental data.

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